Micro C - Spring 2014 - Re-Exam Solutions

1. Consider the following game G:

		Player 2		
		D	E	F
Player 1	A	$7,7$	$1,8$	$1,6$
	B	$2,0$	$3,3$	$2,2$
	C	$6,3$	$2,1$	$5,4$

   (a) Find all pure strategy Nash Equilibria in G. Find a mixed strategy Nash Equilibrium where Player 1 randomizes between B and C and Player 2 randomizes between E and F.

   SOLUTION: The pure strategy Nash Equilibria are (B,E) *and* (C,F). *They can be found by underlining the highest payoff for Player 1 in each row and for Player 2 in each column. To find a mixed strategy equilibrium, suppose that Player 1 randomizes between B (prob.p) and C (prob.1-p), and that Player 2 randomizes between E (prob.q) and F (prob.1-q). For Player 1 to be indifferent between B and C, we require* $3q + 2(1-q) = 2q + 5(1-q)$, *so* $q = 3/4$. *For Player 2 to be indifferent between E and F, we require* $3p + (1-p) = 2p + 4(1-p)$, *so* $p = 3/4$.

   (b) Now consider the game $G(2)$, where G is repeated twice. Find a Subgame Perfect Nash Equilibrium of $G(2)$ where Player 1 earns a total payoff of 12.

   SOLUTION: Consider the following strategy profile. Player 1 plays A in period 1. He plays C in period 2, if the outcome in period 1 was (A,D), *and otherwise he plays B. Player 2 plays D in period 1. He plays F in period 2, if the outcome in period 1 was* (A,D), *and otherwise he plays E. Neither player has an incentive to deviate in period 2, since* (B,E) *and* (C,F) *are both Nash Equilibria of G. Taking into account payoffs in period 2, the strategic situation in period 1 can be represented by the following game (reduced form):*

		Player 2		
		D	E	F
Player 1	A	$12,11$	$4,11$	$4,9$
	B	$5,3$	$6,6$	$5,5$
	C	$9,6$	$5,4$	$8,7$

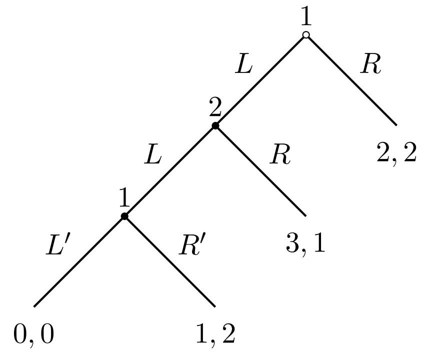
   (A,D) *is a Nash Equilibrium of this game, which means that the original strategy profile for* $G(2)$ *is subgame perfect.*

   (c) Would the Subgame Perfect Nash equilibrium from part (b) still exist if the payoffs $(3,3)$ from strategy profile (B,E) were replaced by $(1,3)$? Explain your answer briefly (2-3 sentences).

   SOLUTION: In this case, the unique Nash Equilibrium of G is (C,F). *This means that the unique Subgame Perfect Nash Equilibrium of* $G(2)$ *has Player 1 always play C and Player 2 always play F. Player 1 earns* $5+5 < 7+5$ *in this equilibrium.*

Players are unable to threaten each other with a 'bad' Nash equilibrium in period 2, which leaves them unable to sustain the 'good' outcome (A,D) in period 1.

2. Now consider this game written in extensive form:



(a) Is this a game of complete or incomplete information?

SOLUTION: By definition, this is a game of complete information.

(b) Write down the strategy sets for Player 1 and for Player 2, and find all pure strategy Subgame Perfect Nash Equilibria.

SOLUTION: $S_1 = \{LL', LR', RL', RR'\}$, $S_2 = \{L, R\}$. *The unique Subgame Perfect Nash Equilibrium is* (RR', L), *which can be found by backwards induction. Player 1 will choose* R' *if play reaches his second decision node, since* $1 > 0$. *This means Player 2 will choose* L *if play reaches his decision node, since* $2 > 1$. *This means Player 1 will choose* R *at his initial decision node, since* $2 > 1$.

(c) Find all pure strategy Nash Equilibria of this game.

SOLUTION: The normal form of this game can be written as

		Player 2	
		L	R
Player 1	LL'	$0, 0$	$3, 1$
	LR'	$1, 2$	$3, 1$
	RL'	$2, 2$	$2, 2$
	RR'	$2, 2$	$2, 2$

The pure strategy Nash Equilibria are (LL', R), (RL', L), (RR', L). *They can be found by underlining the highest payoff for Player 1 in each row and for Player 2 in each column.*

(d) Check that every Subgame Perfect Nash Equilibrium in this game is also a Nash Equilibrium. Briefly explain why this must be the case (2-3 sentences). Take one Nash Equilibrium and describe intuitively why it is not subgame perfect (2-3 sentences).

SOLUTION: In a Subgame Perfect Nash Equilibrium, strategy profiles must constitute a Nash equilibrium in all proper subgames, and also in the game itself. So by definition, every Subgame Perfect Nash Equilibrium is also a Nash Equilibrium. In this game, the Nash Equilibrium (LL', R) *is not subgame perfect. Intuitively, Player 1 makes an incredible threat, by claiming he will play* L', *to convince Player 2 to play*

R. Player 2 should understand that Player 1 would actually play R' if play reached his second decision node, which makes it optimal for Player 2 to play L.

3. Two firms compete in the market for frozen desserts, where Firm 1 produces chocolate ice cream and Firm 2 produces vanilla. These products are imperfect substitutes: demand for chocolate ice cream is $q_1 = 1 - p_1 + p_2$, and demand for vanilla is $q_2 = 1 - p_2 + p_1$, where p_1 and p_2 are the prices of Firm 1 and 2. Firm 1 has access to advanced ice cream production technology that allows it to produce at zero marginal cost. Firm 2 is still using old-fashioned production techniques, so it has marginal costs of $c > 0$. This means profits for Firm 1 are $\pi_1 = q_1 p_1$, and profits for Firm 2 are $\pi_2 = q_2(p_2 - c)$. Firms set prices simultaneously and independently.

   (a) Show that in the Nash Equilibrium of this game, firms set prices $p_1 = 1 + c/3$ and $p_2 = 1 + 2c/3$. Explain intuitively why both equilibrium prices are increasing in c (2-3 sentences).

   SOLUTION: Taking the first-order condition gives best-response functions $p_1 = (1 + p_2)/2$, and $p_2 = (1 + p_1 + c)/2$, which yields equilibrium prices $p_1 = 1 + c/3$, $p_2 = 1 + 2c/3$. Firm 2's optimal price is directly increasing in its cost, since high cost means lower revenues from every sale. Firm 1's optimal price is indirectly increasing in Firm 2's cost, because of strategic complementarities. When Firm 2 sets a high price, this pushes Firm 1 to set a high price as well, since best-response functions are upward sloping.

   (b) Suppose Firm 2 develops an innovation that it hopes will lower production costs. If the innovation works, then Firm 2's marginal costs become zero. If the innovation does not work, then its marginal costs remain at c. Firm 2 knows whether or not the innovation works (so it knows its own marginal cost), but Firm 1 does not. Firm 1 believes there is a probability 1/2 that the innovation works, so that Firm 2 has marginal cost of zero, and a probability 1/2 that it does not work, so that Firm 2 has marginal cost of c. Write down the three best-response functions that, taken together, implicitly define the prices in the Bayes-Nash equilibrium of this game. *(It is sufficient to write down the best-response functions of the players. You do not need to explicitly solve for the equilibrium prices).*

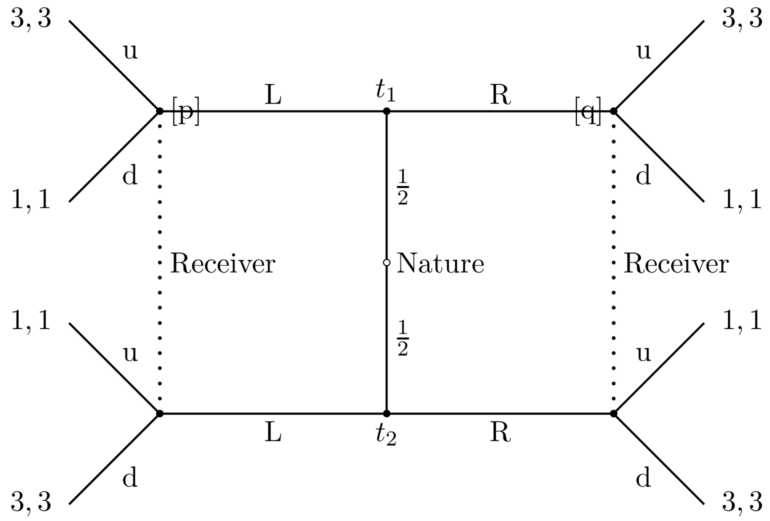
   SOLUTION: From part (a), the best-response function for Firm 2 is $p_2^H = (1 + p_1 + c)/2$ if it a high-cost type. Setting $c = 0$, the best-response function is $p_2^L = (1 + p_1)/2$ if it is a low-cost type. Expected demand for Firm 1 is $(1/2)(1 - p_1 + p_2^H) + (1/2)(1 - p_1 + p_2^L) = 1 - p_1 + (p_2^H + p_2^L)/2$. Its best-response function is $p_1 = (1 + (p_2^H + p_2^L)/2)/2$. Extra: solving these three linear equations in three unknowns gives the equilibrium prices, $p_1 = 1 + c/6$, $p_2^L = 1 + c/12$, $p_2^H = 1 + 7c/12$.

   (c) *(You should attempt to answer this question even if you were unable to solve parts (a) and (b)).* Imagine that before setting prices, Firm 2 announces to Firm 1: "Unfortunately, the innovation does not work, so I have high costs!" Why might Firm 2 make such an announcement? How do you expect this announcement to affect the price set by Firm 1? Explain your answers briefly (2-3 sentences each).

   SOLUTION: Firm 2 would like to convince Firm 1 that it has high costs, since part (a) shows that Firm 1 would then set a higher price. This would increase demand for Firm 2, allowing it to earn higher profits. Firm 1 should understand that Firm

2's announcement is not credible, since it would have an incentive to make this announcement (if it were believed) regardless of its actual costs. In equilibrium, Firm 1 ignores the announcement, Firm 2 realizes it will be ignored, and they set the same prices as in part (b).

4. Consider the following game G':



(a) Is G' a static or a dynamic game?

SOLUTION: By definition, G' is a dynamic game.

(b) Find one separating Perfect Bayesian Equilibrium and one pooling Perfect Bayesian Equilibrium.

SOLUTION: Pooling equilibrium: $(LL, uu, p = 1/2, q = 1/2)$. Separating equilibrium: $(LR, ud, p = 1, q = 0)$. These equilibria are not unique, so other answers may also be correct.

(c) Do these equilibria from part (b) satisfy Signaling Requirement 6?

SOLUTION: Signaling Requirement 6 is trivially satisfied in any separating equilibrium. All information sets are reached with strictly positive probability, so there are no out-of-equilibrium beliefs. Signaling Requirement 6 is also satisfied in any pooling equilibrium of G', since by symmetry, no message is equilibrium dominated.

(d) What real-world strategic situation might correspond to G'? Explain briefly who are the players, what are the messages, what are the actions, and whether you think people are more likely to play a separating equilibrium or a pooling equilibrium (3-5 sentences).

SOLUTION: G' is a cheap talk game where messages are costless. For example, G' might describe a job application process, where the receiver is the firm, the sender is an applicant, and the applicant has private information about which job he is well-suited for. The applicant sends a message saying he is well-suited for position 1 (L) or for position 2 (R). The firm responds by placing him in position 1 (u), or position 2 (d). The payoffs suggest that the interests of the firm and the applicant are aligned *(both benefit if the applicant is placed in a position that matches his skills), so successful communication (a separating equilibrium) would seem likely.*